Climate Economics:

Scientific and Economic Foundations

Take-Home Exam ST2025

at the

Faculty of Business, Economics and Social Sciences
of the University of Berne

lectured by

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Submission date:	18.07.2025

Take Home Exam: Carbon Budget in the Green Solow Model

In the model, produced output net of abatement Y at time t is given by

$$Y(t) = (1-a)F(K(t), T_c(t)L(t)), \tag{1}$$

where a is the constant *abatement share* (i.e., the fraction of GDP used for abatement activities) and F denotes the aggregate production function of the economy

$$F(K(t), T_c(t)L(t)) = \sqrt{K(t)T_c(t)L(t)}, \tag{2}$$

with capital input K, technological level of the conventional sector T_c, and labor input L.

Production also generates greenhouse gas (GHG) emissions E at time t in the amount of

$$E(t) = F(K(t), T_c(t)L(t))C(T_a(t), a), \tag{3}$$

where C denotes the *carbon intensity* (i.e., the amount of GHGs emitted per unit of GDP produced) of the economy and is given by

$$C(T_a(t), a) = \frac{1-a}{T_a(t)}, \tag{4}$$

with $T_a(t)$ denoting the technological level of the abatement sector.

We further assume that capital develops according to the following equation of motion:

$$\dot{K}(t) = sY(t) - \delta K(t), \tag{5}$$

with the constant *savings rate* s (i.e., the fraction of net output invested into the capital stock) and depreciation rate δ.

In addition, we assume that the labor force is constant and normalized to one, i.e., $L(t) = 1$, that the initial capital stock is normalized to one, i.e., $K(0) = 1$, and that the technological levels of the conventional and the abatement sectors are growing at constant rates g_c and g_a, respectively:

$$T_c(t) = T_c(0)\exp(g_c t), \tag{6}$$

$$T_a(t) = T_a(0)\exp(g_a t). \tag{7}$$

Exercise (a)

As given, it holds $L(t) = 1$, and thus, we can write the equations as:

$$y(t) = \frac{Y(t)}{T_c(t)}\,, \tag{8}$$

$$k(t) = \frac{K(t)}{T_c(t)}\,, \tag{9}$$

$$e(t) = \frac{E(t)}{T_c(t)}\,. \tag{10}$$

Using Equation (1), (2) and (6), we can write Equation (8) as:

$$y(t) = \frac{Y(t)}{T_c(t)} = (1-a)\frac{\sqrt{K(t)T_c(t)}}{T_c(t)} = (1-a)\sqrt{\frac{K(t)}{T_c(t)}}$$

and finally, using Equation (9), it leads to

$$(1-a)\sqrt{k(t)}. \tag{11}$$

Reform Equation (6) to get:

$$\dot{T}_c(t) = g_c \cdot T_c(0) \cdot \exp(g_c t) = g_c T_c(t) \quad \Rightarrow \quad \frac{\dot{T}_c(t)}{T_c(t)} = g_c \tag{12}$$

Calculate $\dot{k}(t)$ and simplify:

$$\dot{k}(t) = \frac{d}{dt}\left(\frac{K(t)}{T_c(t)}\right) = \frac{\dot{K}(t)T_c(t)}{T_c(t)^2} - \frac{K(t)\dot{T}_c(t)}{T_c(t)^2} = \frac{\dot{K}(t)}{T_c(t)} - \frac{K(t)}{T_c(t)} \cdot \frac{\dot{T}(t)}{T_c(t)} = \frac{\dot{K}(t)}{T_c(t)} - k(t) \cdot \frac{\dot{T}(t)}{T_c(t)}$$

and using Equation (5) and (12), we get:

$$\dot{k}(t) = \frac{sY(t) - \delta K(t)}{T_c(t)} - g_c k(t) = sy(t) - \delta k(t) - g_c k(t) = sy(t) - (g_c + \delta)k(t).$$

Finally, use Equation (11) to simplify it to

$$\dot{k}(t) = s(1-a)\sqrt{k(t)} - (g_c + \delta)k(t). \tag{13}$$

Finally, being in the steady state ($\dot{k} = 0$), Equation (13) can be reformed, so that we show that the capital stock in terms of effective labor units along the balanced growth path is indeed given by:

$$0 = s(1-a)\sqrt{k^\star} - (g_c+\delta)k^\star \quad \Leftrightarrow \quad s(1-a)\sqrt{k^\star} = (g_c+\delta)k^\star$$

$$\Leftrightarrow \quad s(1-a) = (g_c+\delta)\sqrt{k^\star} \quad \Leftrightarrow \quad k^\star = \left(\frac{s(1-a)}{g_c+\delta}\right)^2. \tag{14}$$

To get the GHG emissions in terms of effective labor units along the balanced growth path, we can insert Equations (1) and (4) into (3) to receive:

$$E(t) = \frac{Y(t)}{T_a(t)}$$

and this can be further used with Equation (11) to get:

$$e(t) = \frac{E(t)}{T_c(t)} = \frac{Y(t)}{T_c(t)T_a(t)} = \frac{y(t)}{T_a(t)} = \frac{1-a}{T_a(t)}\sqrt{k(t)}, \tag{15}$$

Finally, using the the given Equation (7) and the calculated optimal capital k^* from Equation (14), we get the desired term:

$$e^\star(t) = \frac{s(1-a)}{g_c+\delta} \cdot \frac{1-a}{T_a(0)\exp[g_a t]} = \frac{1}{T_a(0)}\frac{s(1-a)^2}{g_c+\delta}\exp[-g_a t]. \tag{16}$$

Exercise (b)

To calculate the balanced growth path of capital K, we use that $k(t) = k^\star$ has to hold, and rearrange Equation (9):

$$K(t) = k^\star T_c(t) \quad \Rightarrow \quad \ln K(t) = \ln k^\star + \ln T_c(t).$$

Differentiate it w.r.t. t, and use in Equation (12) to get:

$$\frac{\dot{K}(t)}{K(t)} = \frac{d}{dt}\ln K(t) = \frac{d}{dt} ln\ k^\star + \frac{d}{dt} ln\ T_c(t) = 0 + \frac{\dot{T}_c(t)}{T_c(t)} = g_c. \tag{17}$$

To calculate the balanced growth path of GHG emissions E, we use that

$$E(t) = \frac{(1-a)}{T_a(t)}\sqrt{K(t)T_c(t)} \quad \Rightarrow \quad \ln E(t) = \ln\,(1-a) + \frac{1}{2}\ln K(t) + \frac{1}{2}\ln T_c(t) - \ln T_a(t).$$

Again, differentiate it w.r.t. t, and use Equations (7), (12), (17) and symmetry:

$$\frac{\dot{E}(t)}{E(t)} = \frac{1}{2}\frac{\dot{K}(t)}{K(t)} + \frac{1}{2}\frac{\dot{T}_c(t)}{T_c(t)} - \frac{\dot{T}_a(t)}{T_a(t)} = g_c - g_a, \tag{18}$$

Using this equation, we can derive that for $g_c < g_a$, the GHG emissions E will decline in the long run.

Exercise (c)

Use the hint to obtain an ordinary first order differential equation in x.

$$\begin{aligned} & x(t) = \sqrt{k(t)} \\ \Rightarrow \quad & k(t) = x(t)^2 \\ \Rightarrow \quad & \dot{k}(t) = 2x(t)\dot{x}(t) \end{aligned} \tag{19}$$

Substitute (13) into the results above to get:

$$\begin{aligned} & 2x(t)\dot{x}(t) = s(1-a)x(t) - (g_c+\delta)x(t)^2 \\ \Leftrightarrow \quad & 2\dot{x}(t) = s(1-a) - (g_c+\delta)x(t) \\ \Leftrightarrow \quad & \dot{x}(t) + \frac{g_c+\delta}{2}x(t) = \frac{s(1-a)}{2}, \end{aligned}$$

Looking like an ordinary first order non-homogeneous differential equation as given in the hint:

$$\dot{x} + \lambda x = b \tag{20}$$

and we can set:

$$\lambda = \frac{g_c+\delta}{2} \tag{21}$$

$$b = \frac{s(1-a)}{2} \tag{22}$$

Solve the homogeneous differential equation to get the general solution with the help of the method of variation of constants:

$$\dot{x}_h + \lambda x_h = 0$$

Then rearrange the terms to:

$$\begin{aligned} \frac{dx_h}{dt} &= -\lambda x_h \\ \frac{1}{x_h}\,dx_h &= -\lambda\,dt \\ \int \frac{1}{x_h}\,dx_h &= \int -\lambda\,dt \\ \ln x_h &= -\lambda t + C_1, \quad \text{where } C_1 \in \mathbb{R} \text{ is a constant} \\ x_h &= \exp(-\lambda t + C_1) \\ x_h &= \exp(C_1)\cdot\exp(-\lambda t) \\ x_h &= A(t)\cdot\exp(-\lambda t) \end{aligned} \tag{23}$$

And differentiate it w.r.t. t:

$$\dot{x} = -\lambda A(t)\text{exp}[-\lambda t] + \dot{A}(t)\text{exp}[-\lambda t].$$

and put it in Equation (20):

$$\begin{aligned}
-\lambda A(t)\exp(-\lambda t) + \dot{A}(t)\exp(-\lambda t) + \lambda A(t)\exp(-\lambda t) = \dot{A}(t)\exp(-\lambda t) &= b \\
\dot{A}(t) &= b\exp(\lambda t) \\
\int \dot{A}(t)\,dt &= \int b\exp(\lambda t)\,dt \\
A(t) &= \frac{b}{\lambda}\exp(\lambda t) + C_2 \quad (24)
\end{aligned}$$

where $C_2 \in \mathbb{R}$ is a constant.

Use Equation (24) in Equation (23) to get:

$$x(t) = \left(\frac{b}{\lambda}\text{exp}[\lambda t] + C_2\right)\text{exp}[-\lambda t] = C_2\text{exp}[-\lambda t] + \frac{b}{\lambda}. \tag{25}$$

Use Equations (21), (22) and (14) from a) to get:

$$\frac{b}{\lambda} = \frac{s(1-a)/2}{(g_c+\delta)/2} = \sqrt{k^\star}. \tag{26}$$

Use that the exercise gives us $K(0) = 1$. Further, normalize Equation (6) to 1 and put it in Equation (9) to get that $k(0) = 1$. Use the hint to get:

$$x(0) = \sqrt{k(0)} = 1$$

For $t = 0$, Equation 25 can be arranged to:

$$\begin{aligned}
1 &= C_2\exp(-\lambda \cdot 0) + \sqrt{k^\star} = C_2 + \sqrt{k^\star} \\
C_2 &= 1 - \sqrt{k^\star} \quad (27)
\end{aligned}$$

Finally, put Equations (26) and Equation (27) in (25) to get:

$$\begin{aligned}
x(t) &= (1-\sqrt{k^\star})\exp(-\lambda t) + \sqrt{k^\star} \\
&= \exp(-\lambda t) + \sqrt{k^\star}(1-\exp(-\lambda t))
\end{aligned}$$

and use Equation (21) and the hint again (Equation (19)) to get

$$k(t) = \left[\exp\left[-\frac{g_c+\delta}{2}t\right] + \sqrt{k^\star}\left(1-\exp\left[-\frac{g_c+\delta}{2}t\right]\right)\right]^2 = x(t)^2 \qquad (28)$$

which is the time path of capital in terms of effective labor. To determine the time paths of emissions in terms of effective labor $e(t)$, simply put Equations (28) and (7) into Equation (15) from a):

$$e(t) = \frac{1-a}{T_a(0)}\left[\exp\left[-\frac{g_c+\delta}{2}t\right] + \sqrt{k^\star}\left(1-\exp\left[-\frac{g_c+\delta}{2}t\right]\right)\right]\exp[-g_a t].$$

Exercise (d)

Rearrange Equation (9) to:

$$K(t) = k(t)T_c(t)$$

Then use this equation with the following from b):

$$\begin{aligned} E(t) &= \frac{1-a}{T_a(t)}\sqrt{K(t)T_c(t)} \\ &= \sqrt{k(t)}\cdot T_c(t)\cdot\frac{1-a}{T_a(t)} \end{aligned}$$

then take the logarithm:

$$\ln E(t) = \frac{1}{2}\ln k(t) + \ln T_c(t) + \ln(1-a) - \ln T_a(t)$$

and finally:

$$\frac{\dot{E}(T)}{E(T)} = \frac{1}{2}\cdot\frac{\dot{k}(T)}{k(T)} + \frac{\dot{T}_c(T)}{T_c(T)} - \frac{\dot{T}_a(T)}{T_a(T)} = g_c - g_a + \frac{1}{2}\cdot\frac{\dot{k}(T)}{k(T)} = 0 \qquad (29)$$

Use Equation (13) from a):

$$\frac{\dot{k}(t)}{k(t)} = \frac{s(1-a)}{\sqrt{k(t)}} - (g_c+\delta).$$

To get $k(t)$, put it in Equation (29)

$$\begin{aligned} 0 &= g_c - g_a + \frac{1}{2}\left(\frac{s(1-a)}{\sqrt{k(T)}} - (g_c+\delta)\right) \\ \frac{s(1-a)}{\sqrt{k(T)}} &= 2g_a - g_c + \delta \\ k(T) &= \left(\frac{s(1-a)}{2g_a - g_c + \delta}\right)^2 \equiv k^T \end{aligned}$$

Then use Equation (28) with (21) to determine T:

$$k^T = \left[\exp(-\lambda T) + \sqrt{k^\star}(1 - \exp(-\lambda T))\right]^2 = \left[\sqrt{k^\star} + (1 - \sqrt{k^\star})\exp(-\lambda T)\right]^2$$

$$\sqrt{k^T} - \sqrt{k^\star} = (1 - \sqrt{k^\star})\exp(-\lambda T)$$

$$\frac{\sqrt{k^T} - \sqrt{k^\star}}{1 - \sqrt{k^\star}} = \exp(-\lambda T)$$

$$T = -\frac{1}{\lambda}\ln\left(\frac{\sqrt{k^T} - \sqrt{k^\star}}{1 - \sqrt{k^\star}}\right)$$

$$T = \frac{1}{\lambda}\ln\left(\frac{1 - \sqrt{k^\star}}{\sqrt{k^T} - \sqrt{k^\star}}\right) \tag{30}$$

Last but not least, determine the level of GHG emissions $E(T)$ at which GHG emissions peak using the term for λ and the hint:

$$\begin{aligned}
E(T) &= \sqrt{k^T} \cdot \frac{T_c(T)(1-a)}{T_a(T)} \\
&= \sqrt{k^T} \cdot \frac{T_c(0)(1-a)}{T_a(0)} \cdot \exp\left[(g_c - g_a) \cdot \frac{1}{\lambda}\ln\left(\frac{1 - \sqrt{k^\star}}{\sqrt{k^T} - \sqrt{k^\star}}\right)\right] \\
&= \sqrt{k^T} \cdot \frac{T_c(0)(1-a)}{T_a(0)} \cdot \left(\frac{1 - \sqrt{k^\star}}{\sqrt{k^T} - \sqrt{k^\star}}\right)^{\frac{2(g_c - g_a)}{g_c + \delta}}
\end{aligned}$$

Exercise (e)

Use this equation from b) again:

$$\begin{aligned}
E(t) &= \frac{1-a}{T_a(t)}\sqrt{K(t)T_c(t)} \\
&= \sqrt{k(t)} \cdot T_c(t) \cdot \frac{1-a}{T_a(t)}
\end{aligned}$$

with Equation (28) and replace some parts with the term equal to λ:

$$E(t) = \frac{T_c(0)(1-a)}{T_a(0)}\left[(1 - \sqrt{k^\star})\exp[-\lambda t] + \sqrt{k^\star}\right]\exp[(g_c - g_a)t]$$

Take the integral on both sides:

$$\begin{aligned}
\int_0^\infty E(t)dt &= \int_0^\infty \frac{T_c(0)(1-a)}{T_a(0)}\left[(1 - \sqrt{k^\star})\exp[-\lambda t] + \sqrt{k^\star}\right]\exp[(g_c - g_a)t]dt \\
&= \frac{T_c(0)(1-a)}{T_a(0)}\int_0^\infty \left[(1 - \sqrt{k^\star})\exp[(g_c - g_a - \lambda)t] + \sqrt{k^\star}\exp[(g_c - g_a)t]\right] dt \\
&= \frac{T_c(0)(1-a)}{T_a(0)}\left[(1 - \sqrt{k^\star})\int_0^\infty \exp[(g_c - g_a - \lambda)t]dt + \sqrt{k^\star}\int_0^\infty \exp[(g_c - g_a)t]dt\right]
\end{aligned}$$

Then, under the same condition as from a), $g_a > g_c$, it leads to:

$$\begin{aligned}\int_0^{\infty} E(t)\,dt &= \frac{T_c(0)(1-a)}{T_a(0)}\left[\frac{1-\sqrt{k^\star}}{g_a-g_c+\lambda}+\frac{\sqrt{k^\star}}{g_a-g_c}\right]\\ &= \frac{T_c(0)(1-a)}{T_a(0)}\cdot\frac{g_a-g_c+\lambda\sqrt{k^\star}}{(g_a-g_c+\lambda)(g_a-g_c)}\end{aligned}$$

Use Equation (14) for k^* and Equation (21) for λ to rewrite the second term:

$$\begin{aligned}\frac{g_a-g_c+\frac{g_c+\delta}{2}\frac{s(1-a)}{g_c+\delta}}{(g_a-g_c+\frac{g_c+\delta}{2})(g_a-g_c)} &= \frac{g_a-g_c+\frac{1}{2}s(1-a)}{(g_a+\frac{1}{2}\delta-\frac{1}{2}g_c)(g_a-g_c)}\\ \frac{2(g_a-g_c)+s(1-a)}{2(g_a+\frac{1}{2}\delta-\frac{1}{2}g_c)(g_a-g_c)} &= \frac{1}{2g_a+\delta-g_c}\left(2+\frac{s(1-a)}{g_a-g_c}\right)\end{aligned}$$

Then we can finally show that the aggregate GHG emissions over time are indeed given by:

$$\int_0^{\infty} E(t)dt = \frac{T_c(0)(1-a)}{T_a(0)(2g_a+\delta-g_c)}\left(2+\frac{s(1-a)}{g_a-g_c}\right). \tag{31}$$

Further, rearrange the equation for a, to calculate the abatement share a for which aggregate emissions stay below a given carbon budget B. To make the calculations easier, set (variables $A-C$ already used):

$$\int_0^{\infty} E(t)\,dt = DX\,(2+EX) = B \tag{32}$$

$$\begin{aligned}\text{where}\quad X &= (1-a)\\ D &= \frac{T_c(0)}{T_a(0)(2g_a+\delta-g_c)}\\ E &= \frac{s}{g_a-g_c}\end{aligned}$$

Then, rearrange Equation (32), so that we can use the quadratic formula:

$$DEX^2+2DX-B=0$$

Use the definition for $X=(1-a)$, to isolate a:

$$\begin{aligned}a &= 1-X\\ &= 1-\frac{-2D+\sqrt{4D^2+4DEB}}{2DE}\\ &= 1-\frac{-D+\sqrt{D^2+DEB}}{DE}\end{aligned} \tag{33}$$

How does the abatement share a depend on international climate policy?

The abatement share a, which determines the proportion of economic activity devoted to reducing greenhouse gas (GHG) emissions, is significantly influenced by international climate policies through three key channels.

First, a more ambitious international climate policy is typically associated with a lower permissible carbon budget B, intended to limit global warming. A smaller carbon budget imposes stricter constraints on cumulative emissions and necessitates greater reductions in emissions over time. This in turn implies that a larger portion of output must be allocated to abatement activities, resulting in a higher required abatement share a.

Second, technological progress in the abatement sector, captured by the growth rate g_a, affects the efficiency and cost of abatement. As g_a increases, abatement technologies become more effective and less costly over time. This reduces the cost of reducing emissions per unit and makes it easier to achieve mitigation targets. Consequently, higher values of a become more economically viable and politically feasible. Therefore, international cooperation to foster innovation and investment in green technologies encourages higher values of g_a, which in turn supports and enables larger abatement shares a.

Third, growth in the conventional (polluting) sector, represented by the technological growth rate g_c, raises the baseline emissions if not mitigated. An increase in g_c means that more abatement effort is required to maintain the same level of cumulative emissions within the budget B. As a result, the abatement share a must increase to offset the rise in emissions. Without sufficient abatement, total emissions could exceed the allowable budget, making climate targets unattainable. Thus, unregulated technological growth in the conventional sector increases pressure on policy frameworks to implement stricter abatement requirements.

As a sidenote, it is important to recognize that international cooperation on climate policy is often far more complex than it appears. While a shared objective—such as limiting global emissions within a specified carbon budget—may be formally agreed upon, aligning the incentives of all participating countries remains challenging. In practice, some actors may strategically over-emit greenhouse gases (GHGs) or reduce their own abatement shares, anticipating that others will compensate by increasing their own abatement efforts.

Exercise (f)

Suppose the following values hold:

$$s = 25\%, \quad \delta = 10\%, \quad B = 400\,[\text{GtC}],$$

$$T_c(0) = 85{,}000\,[10^9\,\text{US/ a}], \quad g_c = 2\%,$$

$$T_a(0) = 8{,}500\,[10^9\,\text{US / (GtC·a)}], \quad g_a = 6\%.$$

Calculate D and E:

$$\begin{aligned}
D &= \frac{T_c(0)}{T_a(0)(2g_a + \delta - g_c)} \\
&= \frac{85{,}000\ [10^9\ \text{US\$}/a]}{8{,}500\ [10^9\ \text{US\$}/(\text{GtC} \cdot a)] \cdot (2 \cdot 0.06 + 0.10 - 0.02)} \\
&= \frac{85{,}000\ [10^9\ \text{US\$}/a]}{8{,}500\ [10^9\ \text{US\$}/(\text{GtC} \cdot a)] \cdot 0.20} \\
&= \frac{85{,}000}{8{,}500 \cdot 0.20}\ \text{GtC} \\
&= \frac{85{,}000}{1{,}700}\ \text{GtC} \\
&= 50\ \text{GtC}
\end{aligned}$$

$$\begin{aligned}
E &= \frac{s}{g_a - g_c} \\
&= \frac{0.25}{0.06 - 0.02} \\
&= \frac{0.25}{0.04} \\
&= 6.25
\end{aligned}$$

Then calculate a (rounded by 5 decimals):

$$
\begin{aligned}
a &= 1 - \frac{-D + \sqrt{D^2 + DEB}}{DE} \\
&= 1 - \frac{-50 \text{ GtC} + \sqrt{(50 \text{ GtC})^2 + 50 \text{ GtC} \cdot 6.25 \cdot 400 \text{ GtC}}}{50 \text{ GtC} \cdot 6.25} \\
&= 1 - \frac{-50 \text{ GtC} + \sqrt{2{,}500 \text{ GtC}^2 + 125{,}000 \text{ GtC}^2}}{312.5 \text{ GtC}} \\
&= 1 - \frac{-50 \text{ GtC} + \sqrt{127{,}500 \text{ GtC}^2}}{312.5 \text{ GtC}} \\
&= 1 - \frac{-50 \text{ GtC} + 357.07142 \text{ GtC}}{312.5 \text{ GtC}} \\
&= 1 - \frac{307.07142 \text{ GtC}}{312.5 \text{ GtC}} \\
&= 1 - 0.98263 \\
&= 0.01737
\end{aligned}
$$

where a denotes the fraction of GDP allocated to abatement activities. In this case, approximately 1.737% of GDP is dedicated to emission reduction efforts.

First calculate λ, $\sqrt{k^*}$ and $\sqrt{k^T}$.

$$
\begin{aligned}
\lambda &= \frac{g_c + \delta}{2} = \frac{0.02 + 0.10}{2} = 0.06 \\
k^\star &= \left(\frac{s(1-a)}{g_c + \delta} \right)^2 \\
\sqrt{k^\star} &= \frac{0.25 \times 0.98263}{0.02 + 0.10} = \frac{0.24566}{0.12} \approx 2.0472 \\
k^T &= \left(\frac{s(1-a)}{2g_a - g_c + \delta} \right)^2 \\
\sqrt{k^T} &= \frac{s(1-a)}{2g_a - g_c + \delta} = \frac{0.25 \times 0.98262}{2 \times 0.06 + 0.1 - 0.02} = \frac{0.24566}{0.2} \approx 1.2283
\end{aligned}
$$

Finally, put it into Equation (30):

$$
\begin{aligned}
T = \frac{1}{\lambda} \ln \left(\frac{1 - \sqrt{k^\star}}{\sqrt{k^T} - \sqrt{k^\star}} \right) &= \frac{1}{0.06} \ln \left(\frac{1 - 2.0472}{1.2283 - 2.0472} \right) = \frac{1}{0.06} \ln(1.27879) \\
\approx 4.09857
\end{aligned}
$$

which is the time T of peak emissions.

Disclaimer

To complete this assignment, I consulted the course materials as well as the paper by Brock and Taylor (2010) Brock and Taylor (2010). In order to refresh my understanding of the course content, I used ChatGPT to generate a structured overview. Additionally, ChatGPT supported me in gaining a deeper understanding of the Brock and Taylor paper, particularly the mathematical sections, which I found challenging.

Throughout the document, ChatGPT was used to assist with coding equations in Overleaf (LaTeX), formatting the layout, and refining the language of my written text to a more academic standard. It also helped check grammar and improve the clarity of my own formulations. Furthermore, I used ChatGPT to cross-check some of my calculations, although I remained cautious, as its results were not always fully accurate.

Subtasks (a) and (b) were solved independently using the provided equations, though I occasionally consulted ChatGPT to cross-check results and to clarify the economic interpretation of certain expressions. For subtask (c), particularly the latter part, I used ChatGPT to clarify the provided hint and to refresh my knowledge of solving differential equations. In subtask (d), ChatGPT helped me interpret the hint and apply it correctly. For subtask (e), the solution to the integral was supported and double-checked using ChatGPT and online integral calculators. ChatGPT also assisted with interpreting the question and formulating the answer in a more formal and academic tone. All calculations, including those in subtask (f), were primarily done manually using paper and a calculator.

References

Brock, William A and M Scott Taylor, "The Green Solow model," *Journal of Economic Growth*, 2010, *15* (2), 127–153.